Multivariate Signature Schemes and Cryptanalysis of Early Proposals

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July 10, 2023

Signature Schemes

Public Key Signature Schemes [Diffie, Hellman, 1976]

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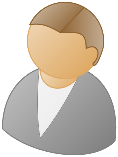

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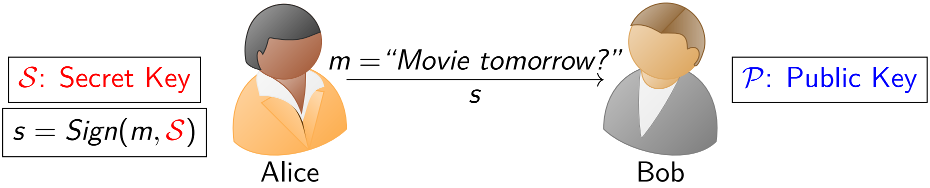

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$\mathcal{S}$: Secret Key

$s = Sign(m, \mathcal{S})$

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$$\xrightarrow[s]{m = \text{“Movie tomorrow?”}}$$

Bob

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$Verify(m, s, \mathcal{P})$

Signature Schemes

Traditional Solutions

- Discrete logarithm (DSA, ElGamal, ECDSA, ...)
- Factoring (RSA)

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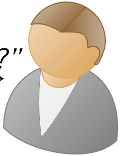

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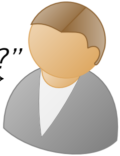

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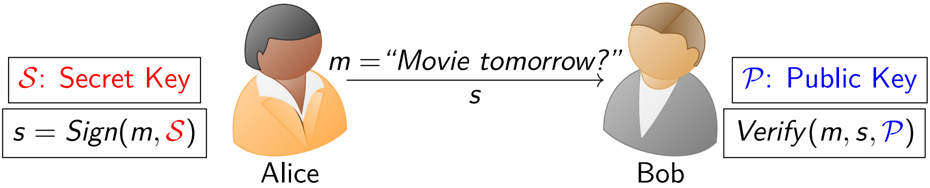
Post-quantum signature schemes?



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Unbalanced Oil and Vinegar, informally [Kipnis, Patarin, Goubin, 1999]

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Multivariate vs Post-Quantum standards

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- NIST Standards: Dilithium, Falcon, SPHINCS+ (**Lattices & Hash**)
- Shorter signatures: suited for **low bandwidth applications**

UOV: Original formulation

Unbalanced Oil and Vinegar [Kipnis, Patarin, Goubin, 1999]

Private Key: - **structured** symmetric matrices $F = (F_1, \dots, F_k)$ in $(\mathbb{F}_q^{n\times n})^k$
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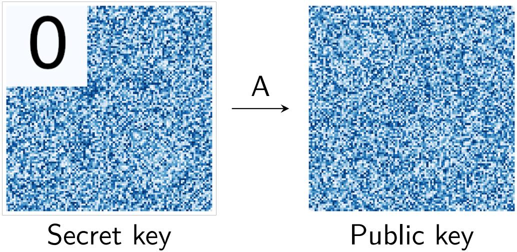

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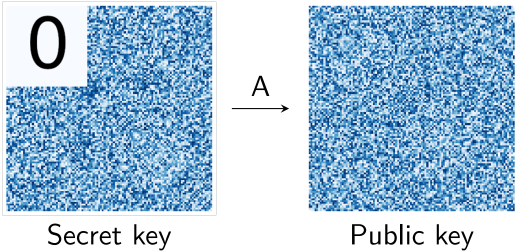

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Private key polynomials: k quadratic forms $\boldsymbol{x}^T F_i \boldsymbol{x}$ **linear** in $x_1, \ldots, x_k$
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In practice: $\boldsymbol{n \leq 3k}$

UOV: Signing process

Signing

A **signature** for the message $\boldsymbol{m} \in \mathbb{F}_q^k$ is a vector $\boldsymbol{x} \in \mathbb{F}_q^n$ such that $1 \leq i \leq k$, $G_i(\boldsymbol{x}) = m_i$

$\boxed{\text{(A,F)}}$

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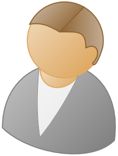

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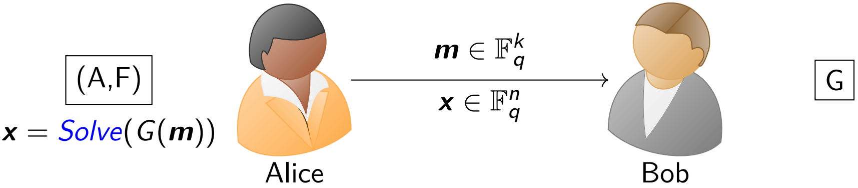

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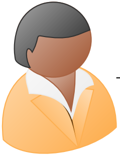
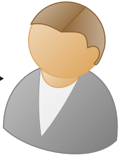
$\boldsymbol{x} = \mathit{Solve}(G(\boldsymbol{m}))$



Alice

$$\xrightarrow[\boldsymbol{x} \in \mathbb{F}_q^n]{\boldsymbol{m} \in \mathbb{F}_q^k}$$



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- Eve forges: $\boldsymbol{x}$ solution of a polynomial system in $x_1, \ldots, x_n$.

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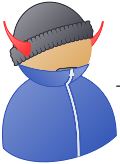
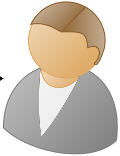


Eve

$$\xrightarrow[\boldsymbol{x} \in \mathbb{F}_q^n]{\boldsymbol{m} \in \mathbb{F}_q^k}$$



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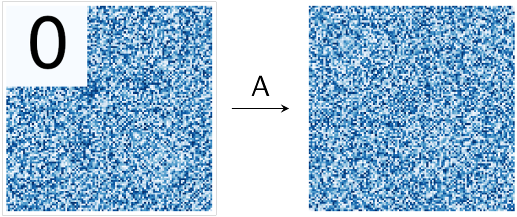
UOV: Alternative formulation

Equivalent characterisation of the trapdoor [Beullens 2020]

Trapdoor: subspace $\mathcal{O}$ of dimension k such that

$$\forall (\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{O}^2, \quad \boldsymbol{x}^T G_1 \boldsymbol{y} = \ldots = \boldsymbol{x}^T G_k \boldsymbol{y} = 0$$



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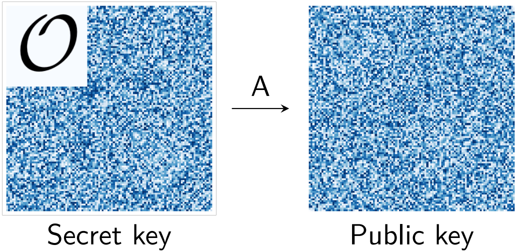

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Forgery

Goal: Find **a** signature $\boldsymbol{x} \in \mathbb{F}_q^n$ for a **single** message $M \in \mathbb{F}_q^k$.

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Computational problem: Find a **linear subspace of dimension** k **in** $V(0)$

Contribution

Input $G, v \in \mathcal{O}$ $\xrightarrow{O(kn^\omega)}$ **Output** $\mathcal{O}$

Main result [P. 2023]

- Polynomial-time algorithm that takes as input **one vector** in $\mathcal{O}$ and the public key G, and returns a basis of $\mathcal{O}$.

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- An attacker needs to find a single vector in $\mathcal{O}$ to retrieve the **secret key** up to equivalence. This is enough to sign **any** message.

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- Finding a vector of $\mathcal{O}$ remains challenging.

State-of-the-art of Key Recovery Attacks

Reconciliation [Ding, Yang, Chen, Chen, Cheng 2008], [Beullens 2020/21]

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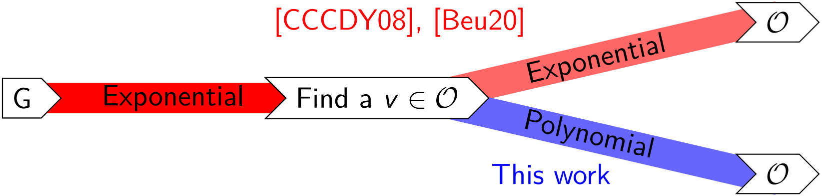
This work

Any vector in $\mathcal{O}$ characterizes it. $\rightarrow$ Polynomial reconciliation



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Observation

$J(\boldsymbol{x})$ is of dimension $n - k$.

Contribution: The algorithm

Public key: $G \in (\mathbb{F}_q^{n \times n})^k$ Secret vector: $\boldsymbol{x} \in \mathbb{F}_q^n$ $\dim(J(\boldsymbol{x})) = n - k$

Reduction

Restriction $G_{|J(\boldsymbol{x})} \to$ UOV instance with **smaller parameters** and one secret vector.

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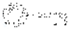

$\longrightarrow \boxed{G, \boldsymbol{x}}$

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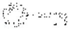

$$\begin{tikzcd} \boxed{G, \boldsymbol{x}} \arrow[d, bend left, "\ker(\boldsymbol{x}^T G_i)"] \\ \boxed{J(\boldsymbol{x})} \end{tikzcd}$$

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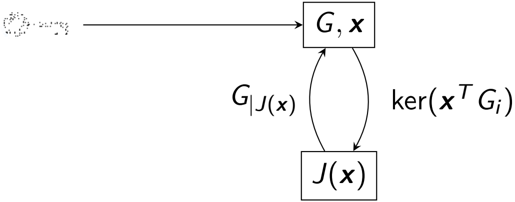

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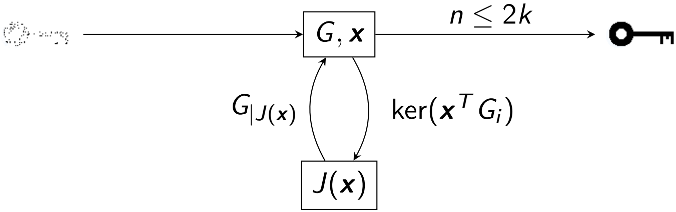

Concluding the attack

$n \leq 2k \to$ broken in **polynomial time**. [Kipnis, Shamir 1998]

Contribution: Complexity analysis

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Complexity of the attack

1. Computing B, a basis of $J(\boldsymbol{x})$ $O(n^\omega)$ and $2 \leq \omega \leq 3$

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2. Computing the restrictions: $G_{i|J(\boldsymbol{x})} = B^T G_i B$ — $O(kn^\omega)$
3. Kipnis-Shamir attack or kernel computations — $O(kn^\omega)$
4. Total cost: $\boldsymbol{O(kn^\omega)}$

Contribution: Experimental results

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ov-Ip	1	112	44	$\mathbb{F}_{256}$	278 432	237 912	43 576	128
ov-Is	1	160	64	$\mathbb{F}_{16}$	412 160	348 720	66 576	96
ov-III	3	184	72	$\mathbb{F}_{256}$	1 225 440	1 044 336	189 232	200
ov-V	5	244	96	$\mathbb{F}_{256}$	2 869 440	2 436 720	446 992	260

Figure: Modern UOV[Beullens, Chen, Hung, Kannwischer, Peng, Shih, Yang 2023]

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Reminder

This is the time it takes, **given** one vector in $\mathcal{O}$, to retrieve a basis of $\mathcal{O}$.

Gap between key recovery and forgery

Key recovery versus forgery

- Experimentally, observe large gap between forgery attacks and key recovery attacks.

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Forgery	0.1s	0.3s	1s	4s	20s	144s	930s	2h	14h
Recovery	40s	1h	2h	>11000h					

Figure: CPU-time in $\mathbb{F}_{31}$ with **msolve** [Berthomieu, Eder, Safey el Din, 2021]

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Goal: forge **a** signature $\boldsymbol{x} \in \mathbb{F}_q^n$ for a **single** message $M \in \mathbb{F}_q^k$.

$$V(M) = \{\boldsymbol{x} \in \mathbb{F}_q^n \quad | \quad \forall i \leq k, G_i(\boldsymbol{x}) = M_i\}$$

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Figure: Implementation of our test $\boldsymbol{x} \in \mathcal{O}$? on a laptop

Perspectives

Contribution [P. 2023]

- One secret vector $\rightarrow$ equivalent easy UOV instance.
- Forgery attack $\rightarrow$ key recovery attack.

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Paper

Preprint to be released, stay tuned!

Thank you for your attention!